Data-Driven Approximations to NP-Hard Problems

Anton Milan  S. Hamid Rezatofighi  Ravi Garg  Anthony Dick  Ian Reid
Motivation

- Learn complex algorithms from data
- Efficient inference
- End-to-end learning
Contributions

1) Sequential (LSTM) bipartite matching
2) Training with “approximate” ground truth
3) Loss- vs. objective-based training
Applications

- **Data Association:**
  Marginalization

- **Keypoint Matching:**
  Quadratic Programming

- **Travelling Salesman Problem:**
  Combinatorics
Applications

- **Data Association:**
  Marginalization

- **Keypoint Matching:**
  Quadratic Programming

- **Travelling Salesman Problem:**
  Combinatorics
Bipartite Matching

Linear Assignment: \[ X^* = \arg\min_X C^T X \quad \text{s.t. } X \text{ binary and one-to-one} \]

\[ \rightarrow \text{ Hungarian (Munkres) Algorithm} \]
Quadratic Assignment Problem

\[ X^* = \arg\max_{X \in \mathcal{X}} J(X) = X^\top K X \]
Our Model
Our Model

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**Bootstrapping the Training Set**

- What if ground truth is ‘hard to obtain’?
- Start with what we have
- Improve over time

![Diagram](image.png)
Bootstrapping the Training Set

- What if ground truth is ‘hard to obtain’?
- Start with what we have
- Improve over time
### Keypoint Matching

<table>
<thead>
<tr>
<th>Name</th>
<th>Accuracy</th>
<th>Objective</th>
<th>Time [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch-and-cut</td>
<td>0.90</td>
<td>10.99</td>
<td>7</td>
</tr>
<tr>
<td>IPFP-S [1] (best of 10) *</td>
<td>0.70</td>
<td>10.47</td>
<td>56</td>
</tr>
<tr>
<td>LSTM</td>
<td>0.76</td>
<td>10.52</td>
<td>4</td>
</tr>
</tbody>
</table>

* Used as ‘ground truth’

[1] Leordeanu et al., 2011
Travelling Salesman Problem

Given a set of nodes, visit each one exactly once and return to start.
Pointer-Networks
[Vinyals et al., NIPS*2015]

- **Loss:** cross-entropy
- **Better:** objective-based training
Objective-based training

Ground Truth

Errors: 0
Length: 310
Objective-based training

Ground Truth

Errors: 0
Length: 310
Objective-based training

Ground Truth

Errors: 0
Length: 310
Objective-based training

Ground Truth

Solution 1

Errors: 0
Length: 310

Errors: 2
Length: 340
Objective-based training

Ground Truth

Solution 1

Errors: 0
Length: 310

Errors: 2
Length: 340
Objective-based training

Ground Truth

Solution 1

Errors: 0
Length: 310

Errors: 2
Length: 340
Objective-based training

<table>
<thead>
<tr>
<th></th>
<th>Ground Truth</th>
<th>Solution 1</th>
<th>Solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Errors:</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Length:</td>
<td>310</td>
<td>340</td>
<td>400</td>
</tr>
</tbody>
</table>
Non-differentiable Loss
Results

**TSP Training**

- Loss-based [21]
- Loss+Objective-based (ours)
- Improved training examples

Tour length vs. Epoch

% Improved train set

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Limitations and Discussion

• Time: Training vs. algorithm design
• Input/Output size is fixed
• Problem’s objective is not always clear (but if it is, use it!)
Conclusions

- LSTM model for matching
- Improving ‘approximate’ training set
- Objective vs. loss-based training
Thank you